7.3 Inversion of z-transform.

$$x[n] = \mathcal{Z}^{-1}\{X(z)\}$$

A) Inspection method.

Ex: $X(z) = \dfrac{1}{1 - az^{-1}}$ $\quad |z| > |a|$

$\therefore\ x[n] = a^n u[n]$

B) Long division method.

Ex: $X(z) = \dfrac{1}{1 - az^{-1}}$ $\quad |z| > |a|$

As the ROC is outside, it is a right sided seq, we will use long division to get the series in powers of z^{-1}

$$\begin{array}{r} 1 + az^{-1} + a^2z^{-2} + \cdots \\ 1 - az^{-1} \overline{)\ 1} \\ \underline{1 - az^{-1}} \\ az^{-1} \\ \underline{az^{-1} - a^2z^{-2}} \\ a^2z^{-2} \end{array}$$

$\therefore\ x[n] = a^n u[n]$

Ex: $X(z) = \frac{1}{1 - az^{-1}}$, $|z| < |a|$

In this we will use long division to get a series of powers of z.

$$
\begin{array}{r|l}
-az^{-1} + 1 & 1 \quad \overline{-az^{-1} - a^2z^{-2} - \cdots} \\
\cline{2-2}
 & -az^{-1} + 1 \\
 & \hline
 & -az^{-1} \\
 & -a^2z^{-2} + az^{-1} \\
 & \hline
 & -a^2z^{-2}
\end{array}
$$

C) Partial fraction.

1) Write $x(z)$ in terms of z.

2) obtain $\frac{X(z)}{z}$

c) If $X(z)/z$ has a numerator degree $\geq$ the denominator degree; the extract the Principle part by polynomial Divisipr and then Do the partial fraction on the rest.

D) Multiply both sides by z and invert term by term.